

## GR16: QUANTUM GENERAL RELATIVITY

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This is the report of the *Quantum General Relativity* session, at the 16th International Conference on General Relativity & Gravitation, held on July 15th to 21st 2001, in Durban, South Africa. This report will appear on the Proceedings of the conference.

### 1 Highlights

Current research in quantum gravity is very lively. The research is actively developing both towards the construction of the fundamental theory and towards applications.

A large part of the efforts to develop the general theory of the quantum gravitational field is presently concentrated in *loop quantum gravity*, in its various forms. The general theory of loop quantum gravity was reviewed at this conference in a plenary talk by Abhay Ashtekar. In the session, several specific aspects of the theory were discussed. The current efforts to understand the low energy behavior of loop quantum gravity were reviewed by Abhay Ashtekar and Jerzy Lewandowski. The covariant spacetime version of the theory, denoted *spinfoam* theory, and some notable finiteness results due also to Alejandro Perez and Louis Crane were illustrated by myself. Various aspects of the discreteness of the geometry predicted by loop quantum gravity were discussed by Seth Major. Unfortunately, Martin Bojowald had to cancel his participation to the conference at the last minute, and his scheduled communication on his exciting recent results on the application of loop quantum gravity to the initial singularity had to be cancelled. The simplicial approach, whose recent developments were reviewed by Ruth Williams, has also recently converged with loop quantum gravity. In fact, on the one hand the spinfoam formalism include the Barrett Crane simplicial model, on the other hand it can be viewed as the covariant version of (canonical) loop quantum gravity. Altogether, loop quantum gravity appears to be in a very exciting phase of intense development and novel results.

Other approaches to quantum gravity were considered in the session as well. In particular: non commutative geometry (Kamesh Wali), the string motivated AdS/CFT theory (Robert Mann), lattice and functional formalisms

(Renate Loll).

The most studied applications of quantum gravity are in black hole physics, gravitational collapse, and cosmological models. The application of quantum gravity to gravitational collapse is a long studied problem. A new and rather surprising results presented by Peter Hájíček, obtained via a careful hamiltonian analysis of the problem came therefore as a surprise: a shell bounces and re-expands, and the singularity disappears. The collapse problem was discussed also by Mike Ryan, Cenalo Vaz and Valentin Gladush.

The most characteristic physical system for quantum gravity is certainly the quantum black hole. There are now two well-known derivations of the Bekenstein-Hawking black hole entropy from a complete quantum gravity theory, obtained respectively within string theory and within loop quantum gravity. But a complete physical understanding of the quantum black hole thermal behavior is not yet available, and the problem keeps raising interest. The problem was discussed in the session by Steven Carlip, Gabor Kunstatter and Louis Witten. Finally, a cosmological model with a perfect fluid was discussed by Julio Fabris.

I'll now illustrate all the contributions to the session in more detail.

## 2 General theory: loop quantum gravity

### ***Abhay Ashtekar: “Semiclassical issues in nonperturbative QG”***

Ashtekar discussed the use of a detailed mathematical theory of quantum geometry, in order to analyze the fundamental discreteness of spacetime predicted by loop quantum gravity. This discreteness makes it difficult to relate the underlying ‘polymer excitations’ of quantum geometry to the Fock states normally used in low energy physics. To bridge this gap, using some recent results of Varadarajan, Ashtekar and Lewandowski have located, analyzed and used Minkowskian Fock states in the background independent framework. The fundamental discreteness is lost in the standard semi-classical description because Fock states constitute only a small subset of all non-perturbative states. While the quantum geometry operators have purely discrete eigenvalues in the full theory, none of these eigenvectors belong to the Fock space. This framework is well-suited to address two key questions: Can the background-independent, non-perturbative theory reproduce the familiar low-energy physics on, say, suitable coarse graining? and, Can one pin-point where and why perturbation theory fails? Ashtekar described work in progress on both these fronts.

**Carlo Rovelli: “Spin foam models”** A spin foam model can be seen as the covariant, or Feynman integral, version of a loop quantum gravity theory. A spin foam represents a four-geometry and the model is formulated as a sum over these four-geometries. Remarkably, there is a duality between these models and certain particular field theories defined on group manifolds (Group Field Theory, or GFT). The sum over spin foam can be generated as the Feynman perturbative expansion of the GFT. Each spacetime appears therefore as the Feynman graph of the auxiliary GFT theory. This phenomenon is a 4d analog of a phenomenon first noticed in 2d quantum gravity, or “zero dimensional string theory”, where the spacetime could be obtained as Feynman graphs of an auxiliary matrix model. The main result presented is that (up to certain degenerate terms) the theory is *finite* order by order in the perturbation expansion. This allows finite three-geometry to three-geometry transition amplitudes to be perturbatively computed explicitly in the physical, lorentzian, 4d theory.

**Seth Major: “Discrete geometry”** Major presented recent results on the spectra of the angle, area, and volume operators, including limits placed on the level of discreteness by semiclassical behavior and classic tests of fundamental physics.

**Ruth Williams: “Progress in discrete quantum gravity”**

Approaches to discrete quantum gravity include sums over histories using the simplicial analogue of the Einstein action formulated by Regge, and state sum models, such as the Ponzano-Regge and Turaev-Viro models in three dimensions and the Barrett-Crane model in four dimensions. There has been a remarkable convergence with the loop quantum gravity approach, in recent years. Williams described progress with simplicial and state sum models, including a discrete version of the calculation of the ground state wave function for linearized gravity, and an investigation of perturbations of three-dimensional state sum models.

### 3 General theory: others

**Kamesh Wali: “Action functionals on a two-sheeted spacetime”**

Wali proposes a model of left- and right- chiral fields living on the two-sheeted space-time along with two distinct gauge fields, within the framework of non-commutative geometry. The mathematics presented generalizes the fundamental concepts of Riemannian geometry, such as differential forms, connection and curvature. The model has a rich

and complex structure with new interaction terms to be explored. One of them is a parity violating interaction arising from the gravitational sector of their generalized theory.

**Robert Mann: “Conserved quantities in AdS/CFT”** The talk reported on work concerning the problem of defining conserved charges in gravitational theory. This is a long-standing problem in general relativity – the equivalence principle implies that locally gravity cannot be distinguished from acceleration, and the problem of defining a local measure of gravitational energy and angular momentum has remained elusive. A reasonable measure of success has been attained in recent years using the Brown/York quasilocal formalism, which proposes a definition of conserved quantities enclosed by a  $(d - 2)$  surface in  $d$  dimensions. A generic problem with the quasilocal formalism is that the conserved quantities typically diverge as the mean radial size of the enclosed surface becomes infinite. A recent proposal inspired by the AdS/CFT correspondence offers a potential resolution to this difficulty. This conjectured correspondence states that a quantum theory of gravity whose boundary conditions are the same as those of asymptotically anti de Sitter (AdS) spacetime is in 1-1 correspondence with a quantum conformal field theory (CFT) defined on the boundary of this spacetime. Quantum field theories generically have counterterms, and the correspondence suggests that these counterterms should be geometric invariants of the induced metric on the boundary at infinity. Employing this proposal, Mann computed the energy, angular momentum and entropy of spacetimes containing rotating black holes with NUT charge (ie in the general Kerr-NUT-anti de Sitter class). He found in all cases that the conserved quantities were finite. No reference spacetime was required at any stage of the calculation. He then showed how to extend this prescription to asymptotically flat spacetimes, and obtained corresponding results that are finite. The interpretation of these quantities in spacetimes with NUT charge is still obscure.

**Renate Loll: “The conformal factor problem”** Under certain assumption on the behavior of the partition function under renormalization, one finds the interesting result that the well known divergence due to the conformal modes is cancelled by a Faddeev-Popov determinant. This result confirms that absence of the conformal sickness noticed by Loll in the lorentzian lattice models.

## 4 Gravitational collapse

**Peter Hájíček: “Gravitational collapse”** Hájíček studies the quantum dynamics of a self-gravitating null shell for the spherically symmetric case, using only gauge invariant quantities (Dirac observables). The definition of evolution is based on an asymptotically non-trivial symmetry of the model (time translation element of the BMS group). Hájíček proves in particular, that a version of quantum theory exists, in which the shell bounces and re-expands, and in which the singularity disappears. The quantum metric outside the shell contains a time-dependent mixture of black and white holes. These remarkable and surprising results raised much interest and a lively discussion, in the session.

**Michael Ryan: “Quantum collapse of dust shell: solutions”** A hamiltonian formulation of the problem of the collapse of a dust shell given by Hájíček and Kijowski and Kuchař allows one to study the minisuperspace problem of the quantum collapse of a spherically symmetric dust shell, albeit with a very complicated Hamiltonian operator. Ryan presented both analytic approximate solutions and exact numerical solutions to the problem. Since the solutions involve only geometrical variables defined on the shell, it is necessary to reconstruct the quantum spacetime geometry around the shell (possible in this special spherically symmetric case) in order to discuss the interpretation of the solutions in terms of spacetime geometry, especially the position of the shell with respect to horizons. This allowed Ryan to consider the quantum evolution of the shell in spacetime.

**Cenalo Vaz: “LeMaitre-Tolman-Bondi collapse”** Vaz applied the Kuchar transformation to LeMaitre-Tolman-Bondi models of inhomogeneous dust collapse. Dirac’s constraint quantization leads to a simple Wheeler-DeWitt equation with a general mass function. A solution was presented over the entire class of models considered. These solutions cover collapse into both black holes and naked singularities (up to the Cauchy Horizon) and can be employed in the study of quantum gravitational effects in the final stages of collapse with different initial conditions as well as, it is hoped, in the study of inhomogeneous cosmologies.

**Valentin Gladush: “Quasiclassical spherical configuration”**  
Gladush has considered a quasi-classical model of self-gravitating

spherical dust shell, and studied the energy spectrum using quasi-classical quantization rules, and the stability of the system.

## 5 Black holes

### ***Steven Carlip: “Boundary conditions, constraints, BH entropy”***

Recent work by a number of authors has suggested that black hole entropy may have a microscopic description in terms of a boundary conformal field theory, living either near the horizon or at spatial infinity. Carlip described a new approach to boundary conditions near such a boundary, in which the fall-off requirements are introduced as a set of second class constraints that modify the algebra of diffeomorphisms. For Liouville theory in a dynamical background, this approach leads to the standard central charge; for dilaton gravity, the result is a Virasoro algebra with a classical central charge of the form required to explain the Bekenstein-Hawking entropy.

***Gabor Kunstatler: “Quantum Mechanics of Charged BHs”*** Basic properties of black hole thermodynamics were used to quantize generic charged black holes in the Euclidean sector. The analysis led to a discrete spectrum for the horizon area that is qualitatively consistent with the results of Bekenstein, Mukhanov and others, but different from the spectrum predicted by loop quantum gravity. In particular, the area spectrum was shown generically to be equally spaced. Quantization of the charge sector gave the condition  $Q = me$ , for integer  $m$ , as expected, but a consistency condition constrained the fine structure constant  $e^2/\hbar$  to be a rational number. Near extremal black holes were shown to be highly quantum objects. Moreover, with a standard choice of factor ordering, extremal black holes did not appear in the physical spectrum at all.

***Louis Witten: “BH Quantization in Canonical Gravity”*** Witten started from the observation that the Schwarzschild black hole can be viewed as a special case of the marginally bound LeMaitre-Tolman-Bondi models of dust collapse corresponding to a constant mass function. Using a midi-superspace quantization of the collapse for an arbitrary mass function illustrated in this same session by C Vaz, Witten showed that the solution leads both to Bekenstein’s area spectrum for black holes as well as to the black hole entropy. In this context, the entropy is naturally interpreted as the loss of information of the original matter distribution within the collapsing cloud.

## 6 Finite dimensional models

**Julio Fabris:** “*Quantum cosmological perfect fluid*” Fabris quantized the gravity-perfect fluid system, employing the Schutz’s formalism. The matter degrees of freedom allow to recover the notion of time. Wave-functions are obtained in closed form, in the mini-superspace, and the expectation value for the scale factor is computed, yielding a singularity-free, bouncing Universe. He showed that these results are recovered by a classical system where, besides the ordinary matter employed in the quantization procedure, a universal repulsive fluid with a stiff matter equation of state is added.